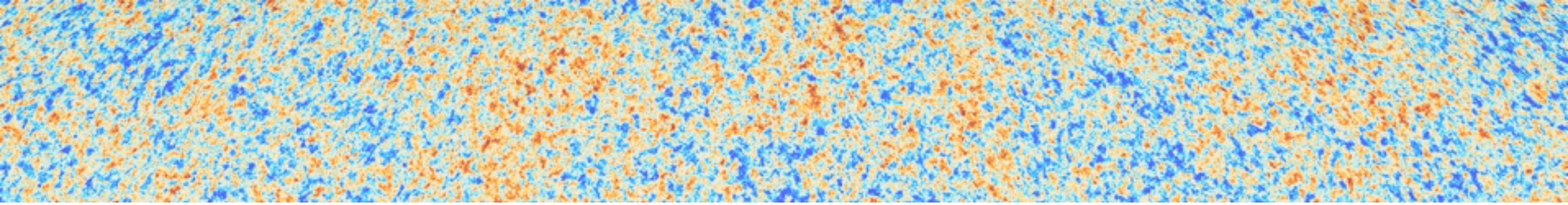


Cosmic Microwave Background
Lecture I

S. Galli

KICP-University of Chicago

ISAPP, Paris, 17 June 2015

Lectures plan

LECTURE I

- A first overview
  - A very short historical introduction
  - CMB maps: monopole, dipole, anisotropies
  - CMB statistics

LECTURE II

- CMB physics (temperature)
  - Recombination and decoupling
- CMB physics (temperature) (cont'd.)
  - Initial conditions and inhomogeneities, from inhomogeneities to anisotropies
- CMB polarization
  - Map characterization
  - Polarization physics

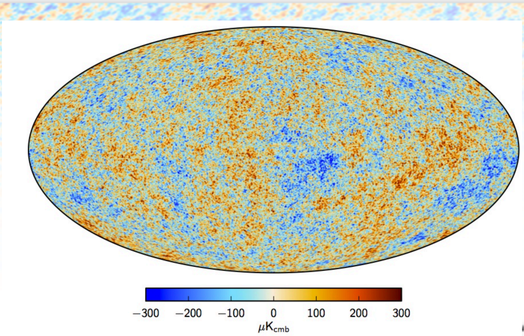
LECTURE III

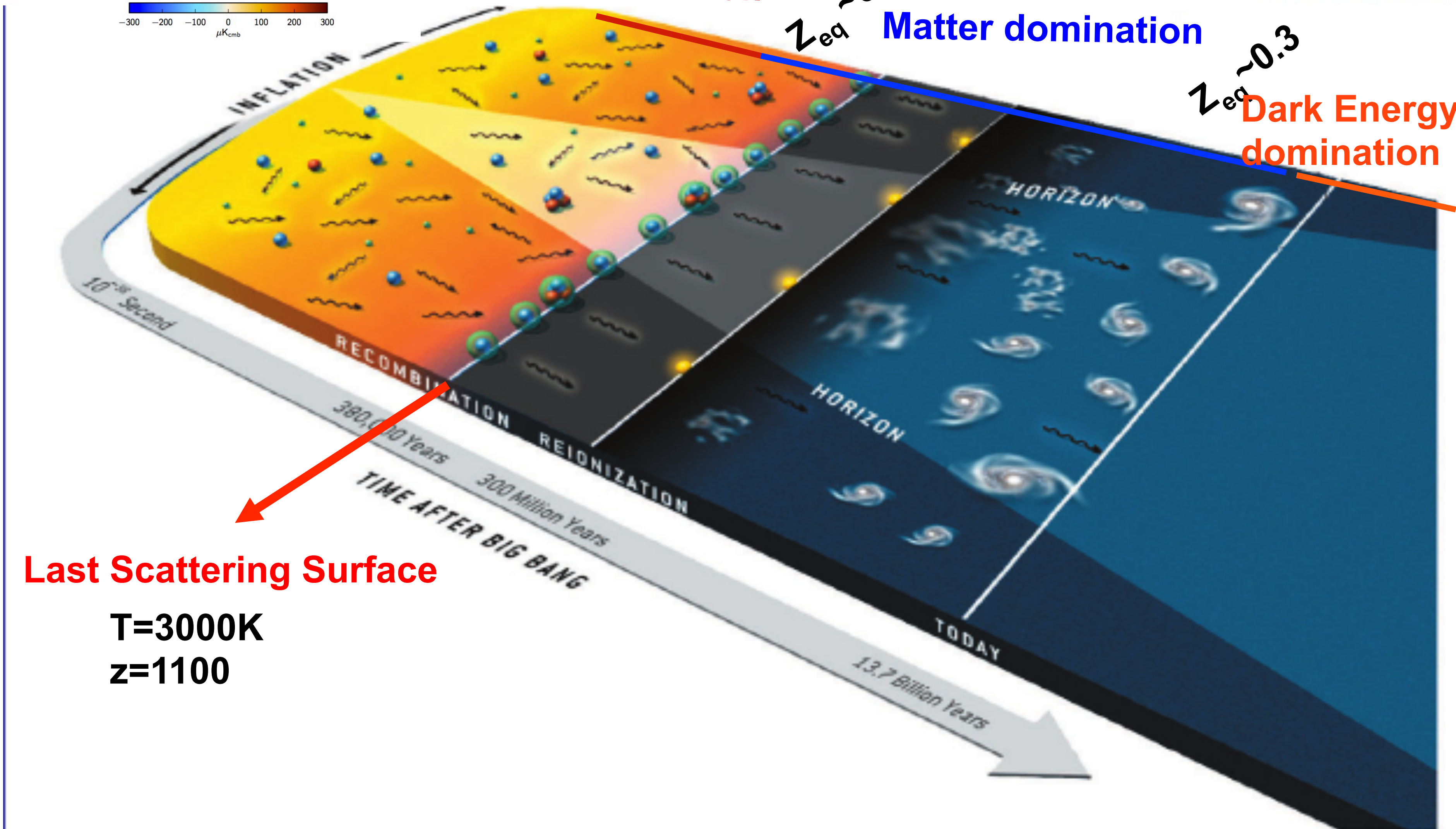
- How to compare theory and data: likelihood!
- A practical example: the Planck satellite

The CMB

- It is an image of the universe at the time of recombination (baryon-photon decoupling), when the universe was just a few hundred thousands years old (z~1100).
- The CMB frequency spectrum
  is a perfect blackbody at T=2.725 K:
  confirmation of the hot big bang model!

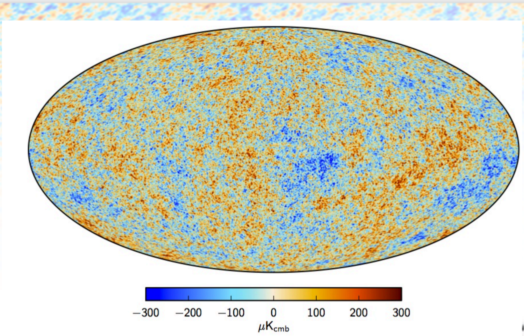
Cosmic History

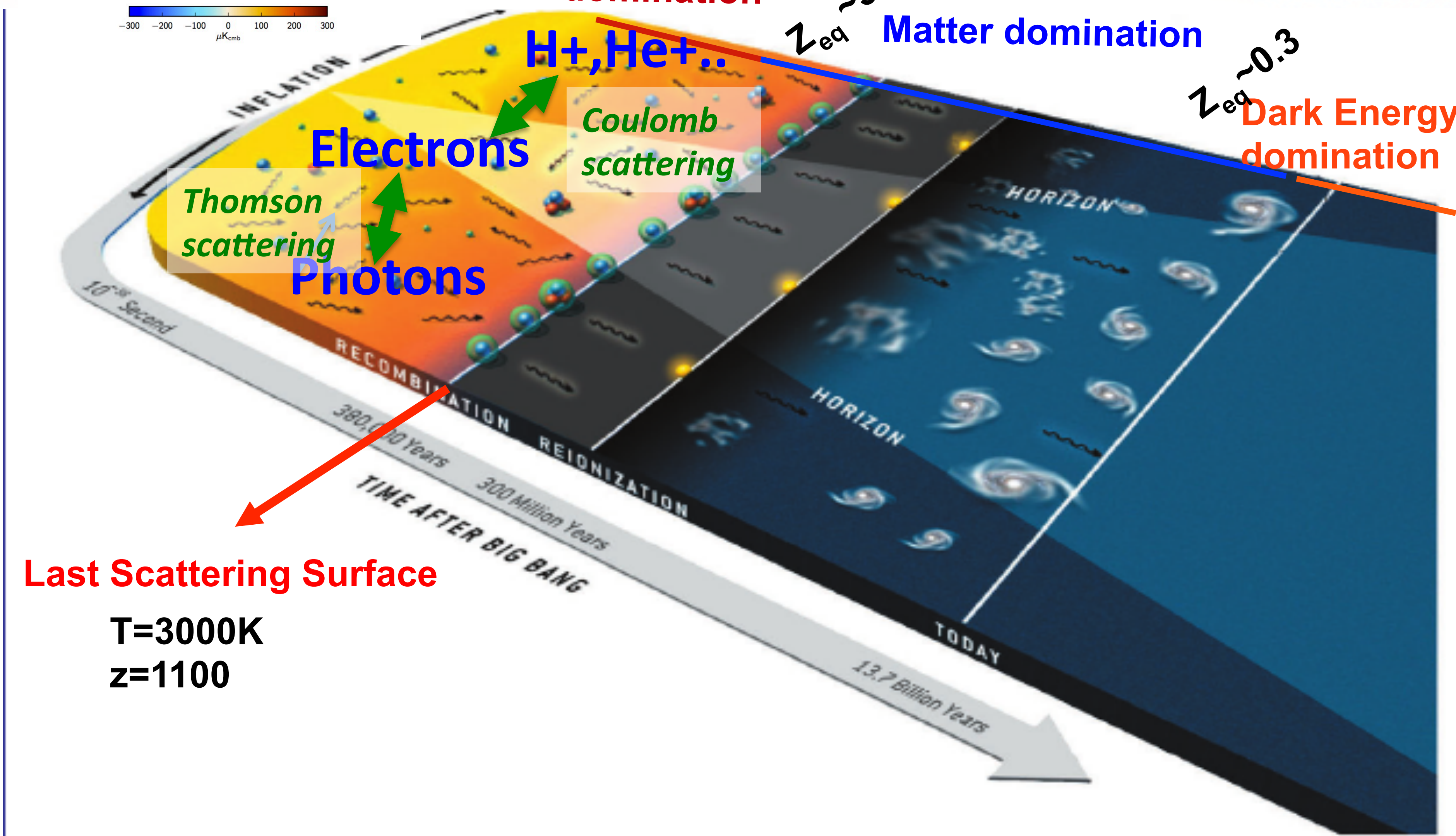




Hu & White (2004); artist: B. Christie/SciAm; available at http://background.uchicago.edu

Cosmic History





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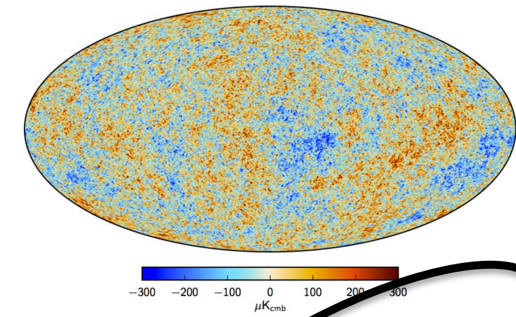
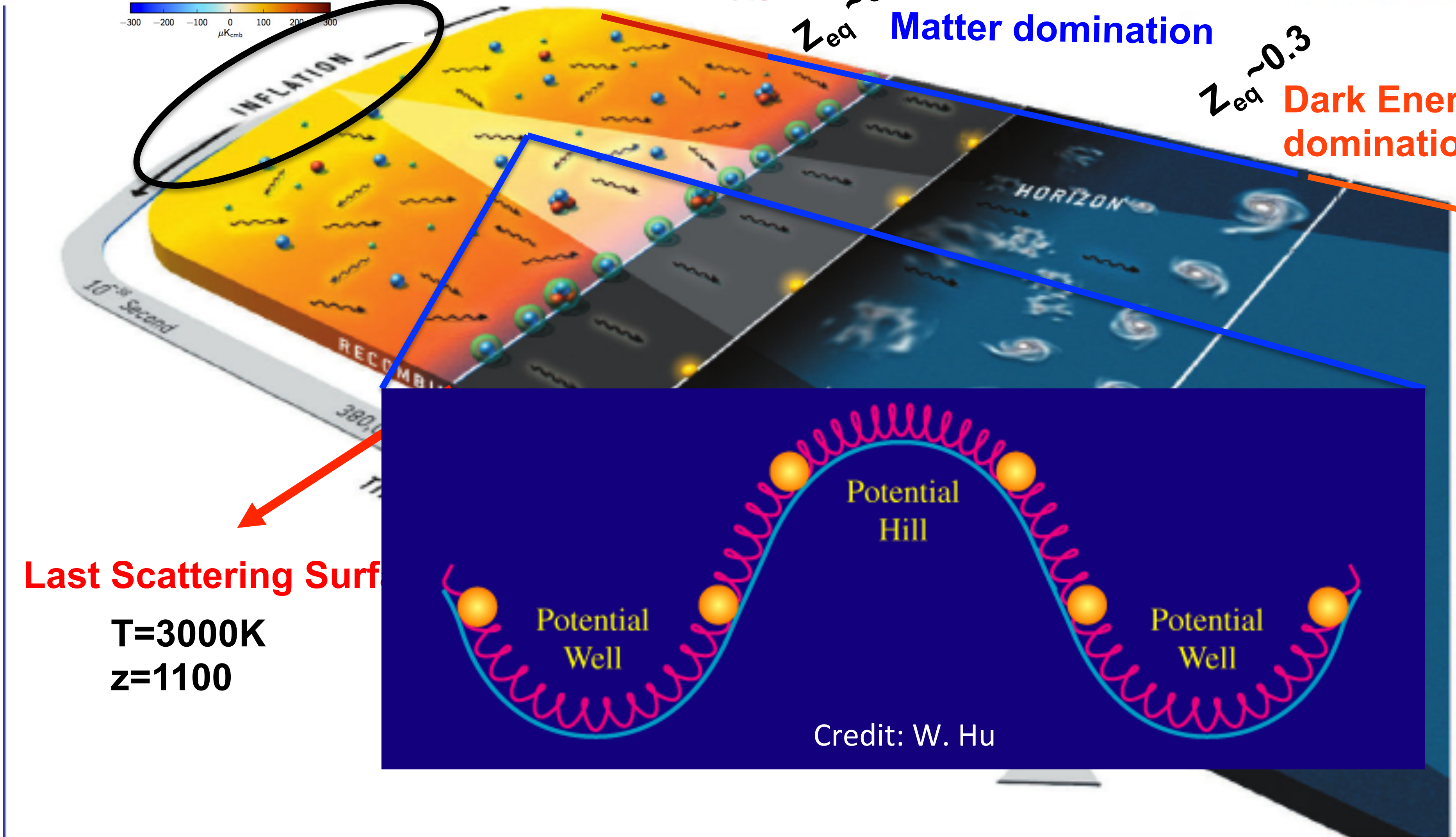
Cosmic History





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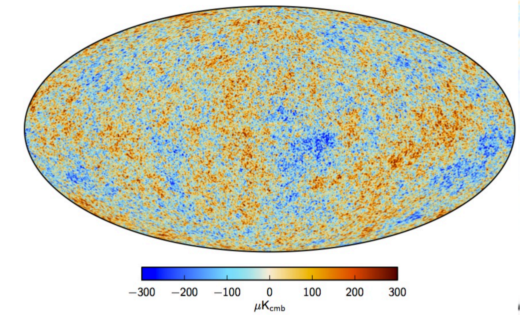
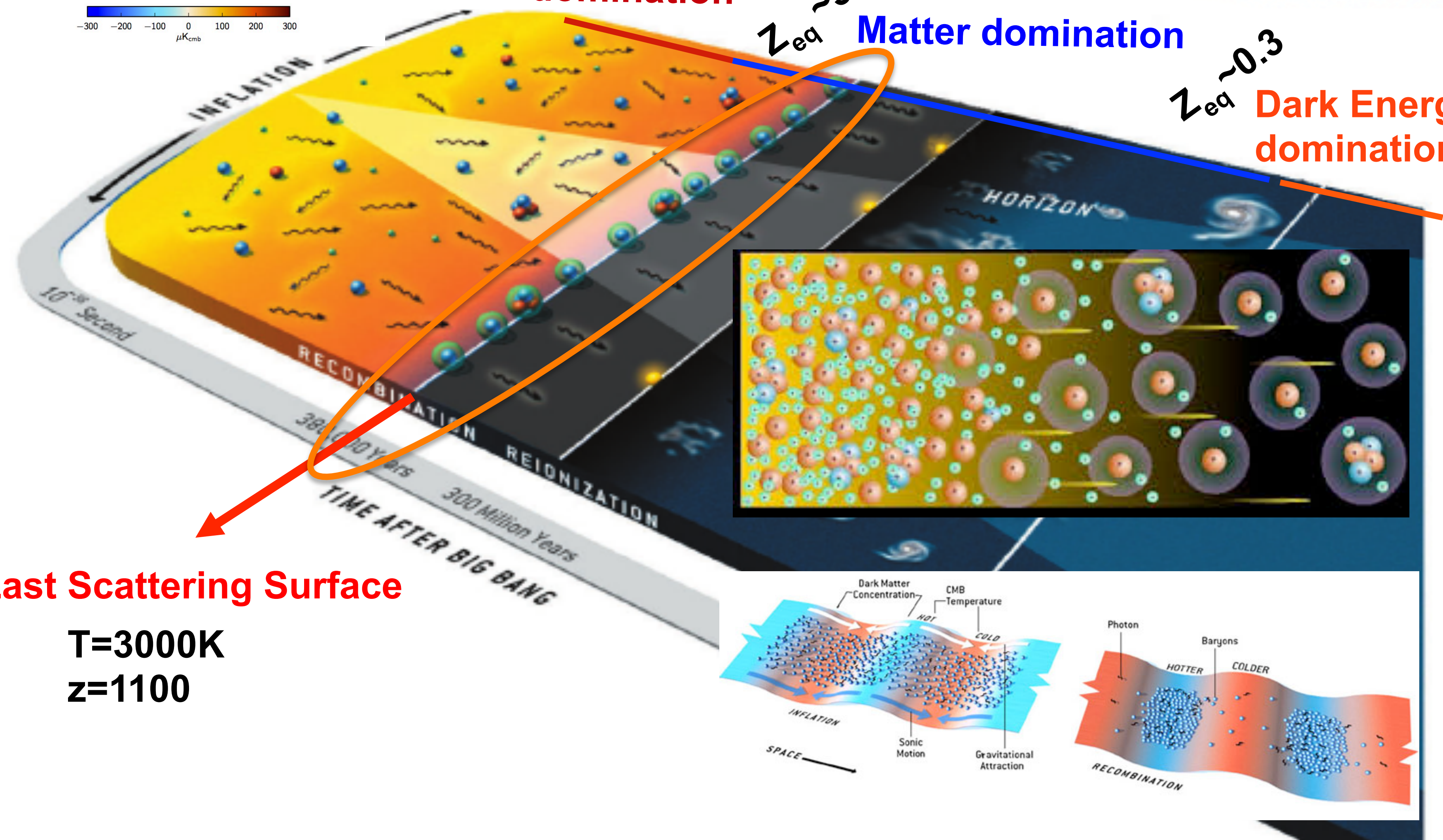
Cosmic History



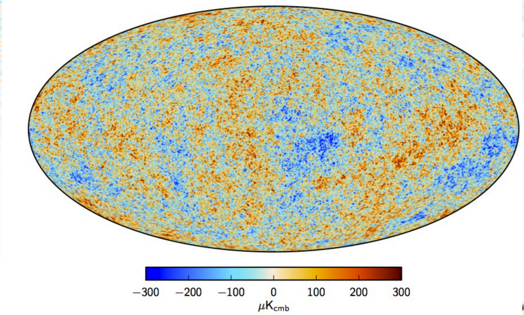


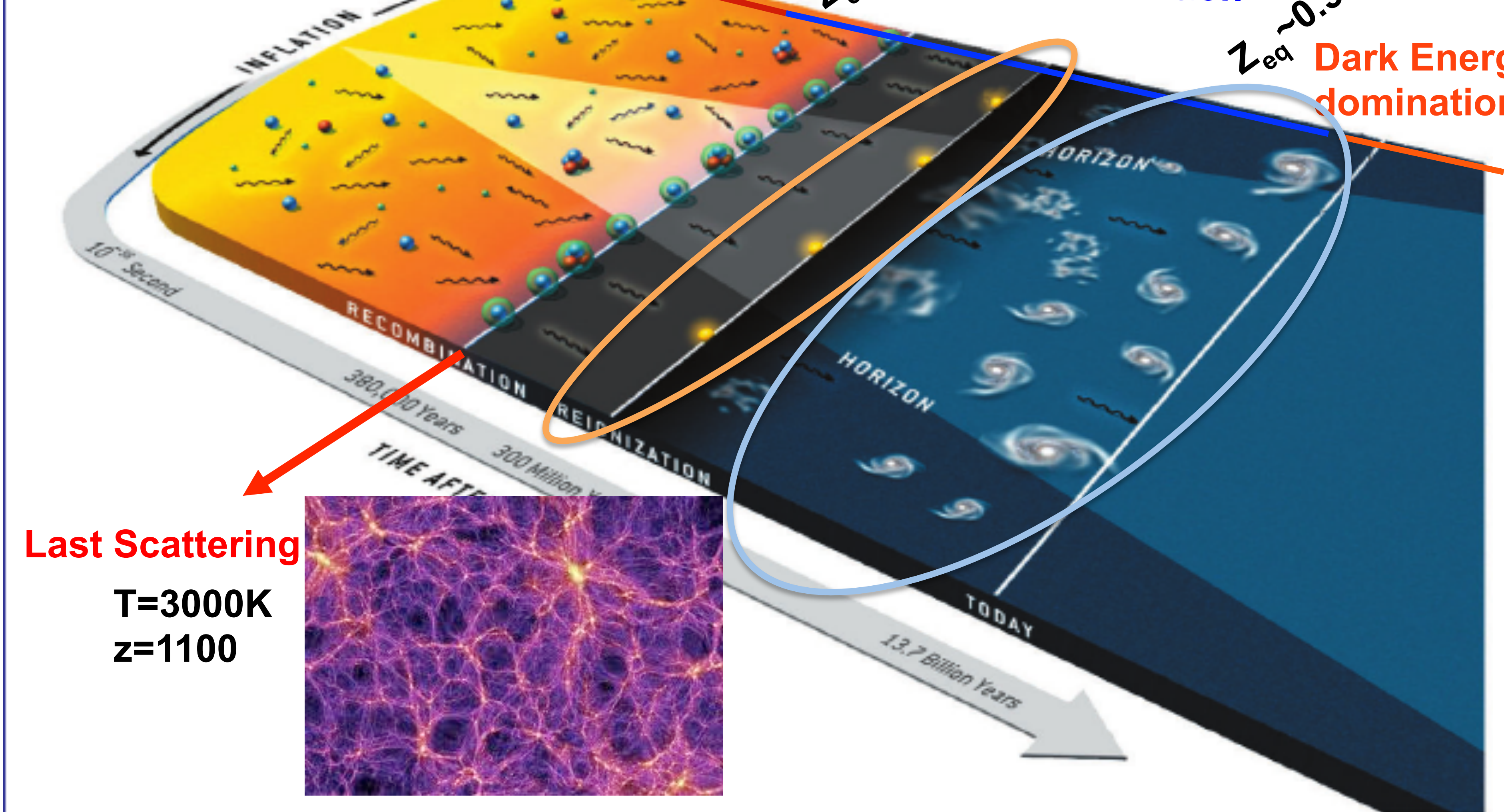
Last Scattering Surface

T=3000K
z=1100

Hu & White (2004); artist: B. Christie/SciAm; available at http://background.uchicago.edu

Cosmic History





Hu & White (2004); artist: B. Christie/SciAm; available at http://background.uchicago.edu

A LITTLE BIT OF HISTORY

A (very) short history of modern cosmology

- 1917 Albert Einstein: Matter bends space. Need a cosmological constant to keep universe steady?

- 1922 Aleksandr Friedman: Universe could be expanding (1) Solution to Einstein's equations in expanding universe.

- 1927 Georges Lamaitre: Universe could be expanding (2), Expansion can cause redshift of galaxies (Hubble's law). First idea of an initial « creation » event.

- 1929 Edwin Hubble: Universe is expanding!Measured redshift of galaxies and their distance (through cepheids).

The Hot Big Bang

- 1946-48 George Gamow: hot Big Bang. Early universe was hot, radiation dominated over matter.
- 1948: Ralph Alpher,Hans Bethe and George Gamow: Primordial nucleosynthesis
- 1948: Alpher and R. Herman: prediction of CMB (at 5K)
- 1964 Doroshkevich and Novikov: detectability of CMB as a proof of Big Bang

The Origin of Chemical Elements

R. A. ALPHER*
Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland

AND

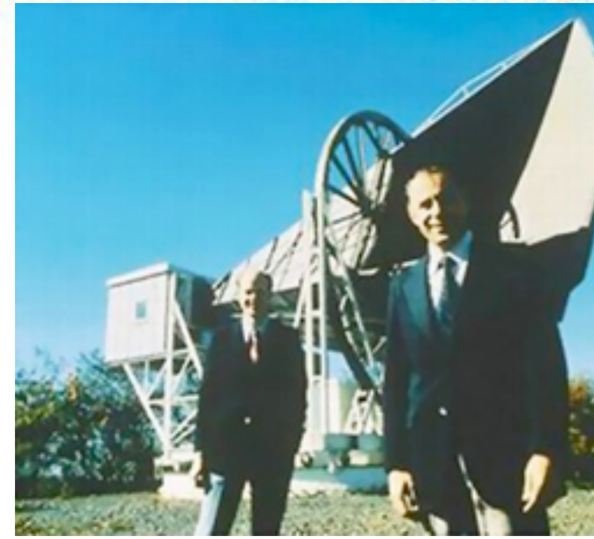
H. BETHE
Cornell University, Ithaca, New York

AND

G. GAMOW
The George Washington University, Washington, D. C.
February 18, 1948

The term "Big Bang" was first used by F. Hoyle at a BBC radio broadcast in 1949 .

Discovery



- 1965: Arno Penzias and Robert Wilson, radio astronomers at Bell Labs in Crawford, New Jersey. Microwave horn radiometer first used for telecommunications, then for astronomy.
- Found uniform noise source (birds nests in the horn?!). From the sky!
- Princeton group (Jim Peebles,Robert Dicke, Peter Roll, and David Wilkinson) was working on CMB detection. Princeton group confirmed Penzias and Wilson discovery.

A MEASUREMENT OF EXCESS ANTENNA TEMPERATURE AT 4080 Mc/s

A. A. Penzias
R. W. Wilson

Astrophysical Journal, vol. 142, p.419-421

COSMIC BLACK-BODY RADIATION*

Astrophysical Journal, vol. 142, p.414-419

R. H. Dicke
P. J. E. Peebles
P. G. Roll
D. T. Wilkinson

Earlier detections?

- 1940 Andrew McKellar observed excited rotational states of CN molecules in interstellar absorption lines. In thermal equilibrium at T~2.3K (see also W. Adams 1941)
- 1955 Émile Le Roux: survey at λ = 33 cm (Nançay Radio Observatory). Near-isotropic background at 3±2K (Denisse, Lequeux, Le Roux 1957, Le Roux PhD thesis 1957).

SEANCE DU 17 JUIN 1957.

RADIOASTRONOMIE. — *Nouvelles observations du rayonnement du Ciel sur la longueur d'onde* 33 cm. Note de MM. **Jean-François Denisse, James Lequeux** et **Émile Le Roux**, présentée par M. André Danjon.

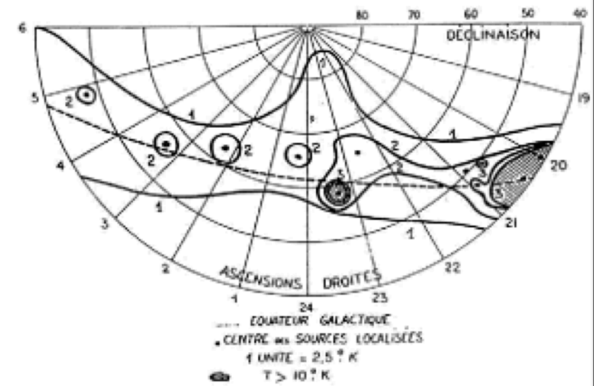


Fig. 1.

Rayonnement du Ciel. — En dehors des régions que l'on vient de décrire, la brillance du Ciel paraît uniforme. Son rayonnement est difficile à mesurer car on l'observe superposé aux émissions beaucoup plus intenses de l'environnement et au bruit propre du récepteur. Nos mesures ont toutefois permis de montrer que la température de brillance du Ciel est inférieure à 3° K et que ses variations d'un point à un autre sont inférieures à 0,5° K.

Reproduction of page 50 of (Leroux's thesis).

En résumé, on a trois équations donnant T_c :

$$v_0 = 0 \qquad 137 = 138 - 0,485\ T_c \longrightarrow T_c = 2°K$$
$$(25) \qquad v_0 = 5 \qquad 50 = 51,3 - 0,485\ T_c \longrightarrow T_c = 2,7°K$$
$$v_0 = -3 \qquad 215 = 218 - 0,77\ T_c \longrightarrow T_c = 3,9°K.$$

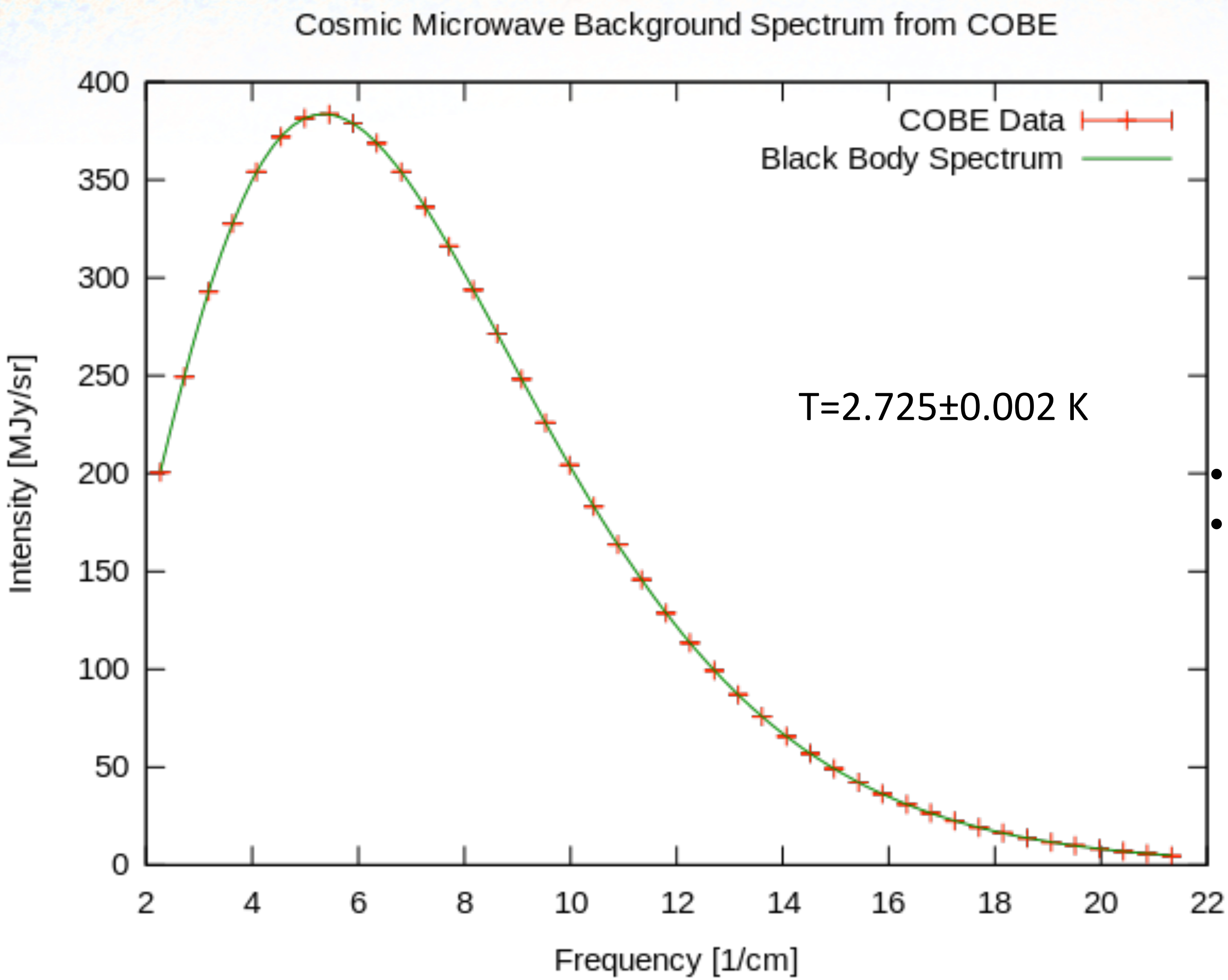
En fait, on devrait déduire, de plusieurs équations de ce genre, les coefficients 1/k, ρ, ρ', ρ'' et T_c. Mais la bonne cohérence des valeurs obtenues pour T_c montre que les valeurs prises pour ces coefficients sont correctes avec une bonne approximation. Si on diminuait le coefficient 1/k on obtiendrait des valeurs négatives pour T_c, quelles que soient les valeurs prises pour ρ' et ρ'' qui interviennent de façon différente dans les 3 équations précédentes, le coefficient ρ' intervenant notamment de façon opposée dans les deux dernières équations. De même, une augmentation de 1/k de quelques pour cent donnerait des valeurs de T_c incohérentes. Enfin, un coefficient de réflection du sol non nul donnerait $T_c < 0$.

Il est difficile de déterminer l'erreur sur cette valeur de T_c, basée sur la cohérence de différentes mesures. Nous pensons que l'erreur absolue doit être de l'ordre de 2° K, en prenant :

$$\boxed{T_c = 3°K}$$

Reproduction in F. Melchiorri,Y. Rephaeli lectures in Varenna 2004

The most accurate measurement to date: COBE



- Launched in 1989.
- Three instruments:

-FIRAS (BB spectrum) [60-2880GHz],1yr

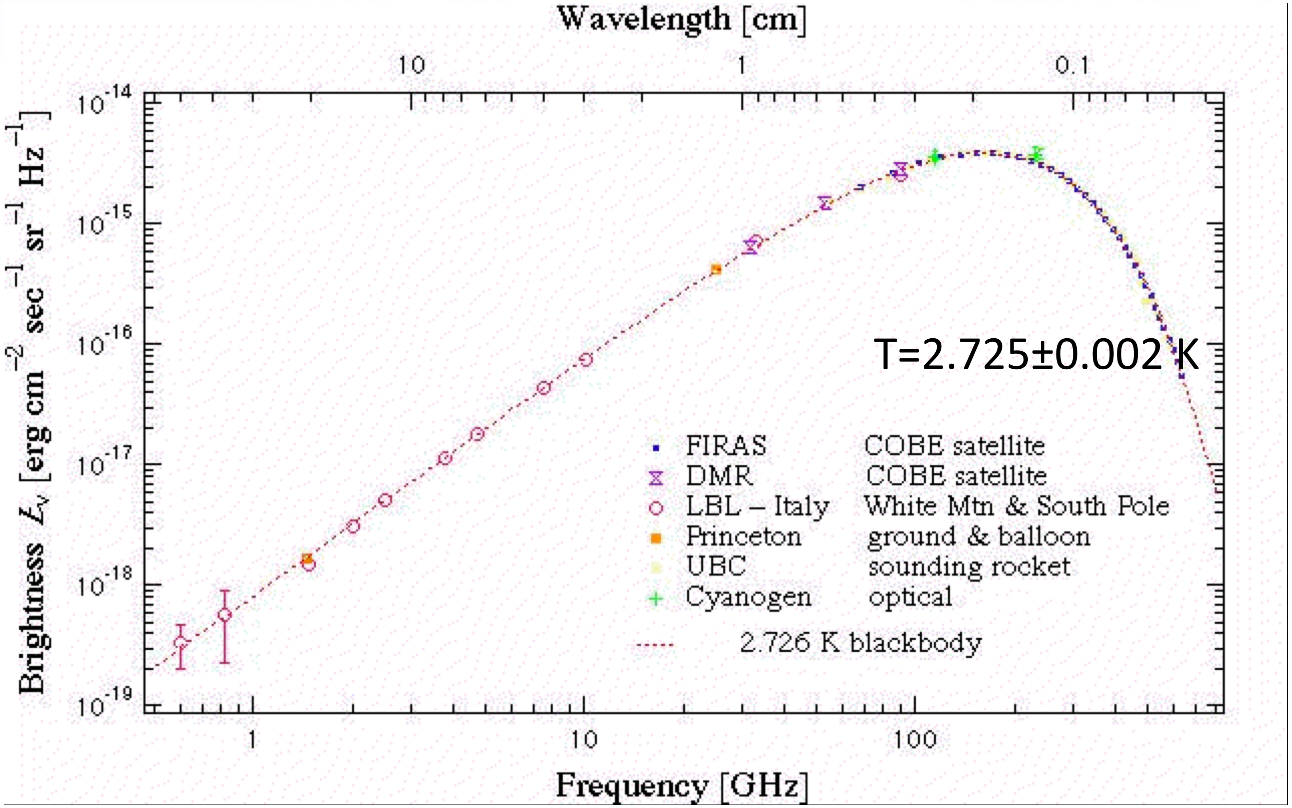
-DMR (anisotropies) [31.5,53,90GHz],4yr

-DIRBE (CIB) [infrared]

FIRAS measurements. Mather et al. (1994, 1996), Fixten 1996

Peak BB(ν) at ~160GHz.

The most accurate measurement to date: COBE

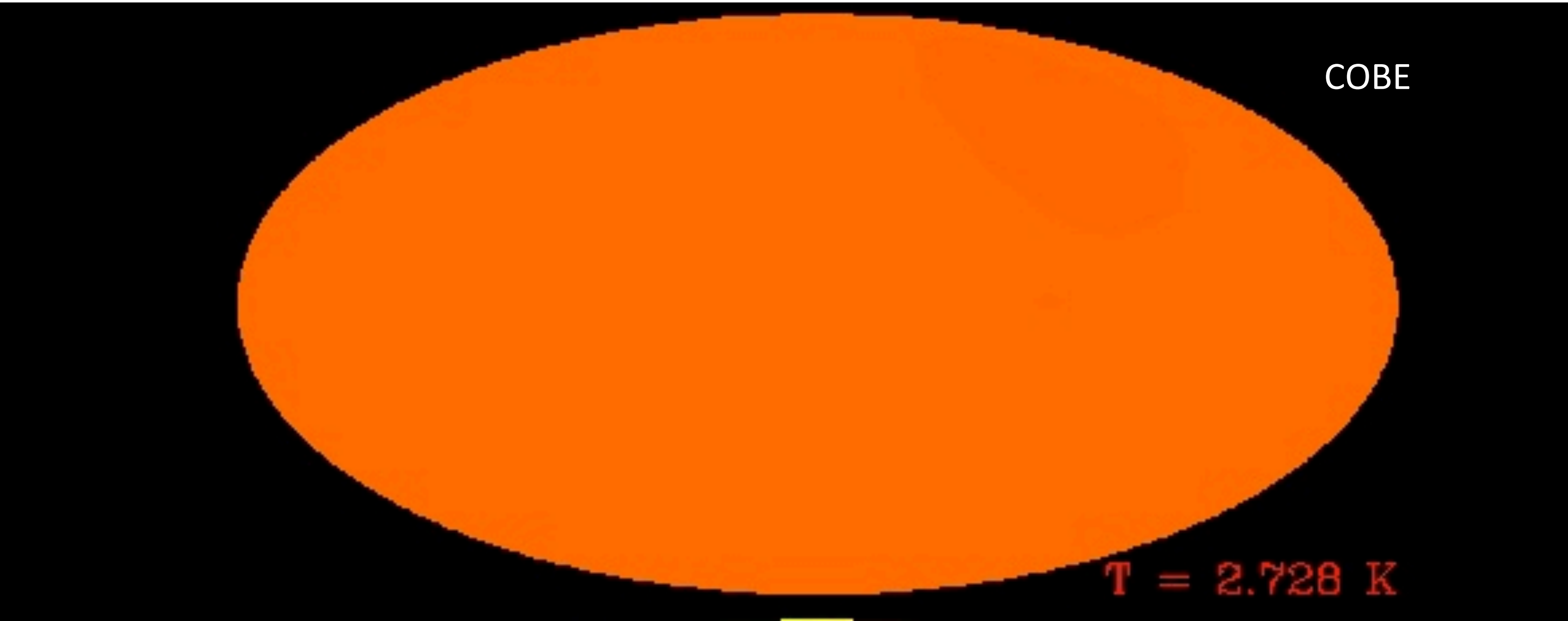


FIRAS measurements. Mather et al. (1994, 1996), Fixten 1996

Peak BB(ν) at ~160GHz.

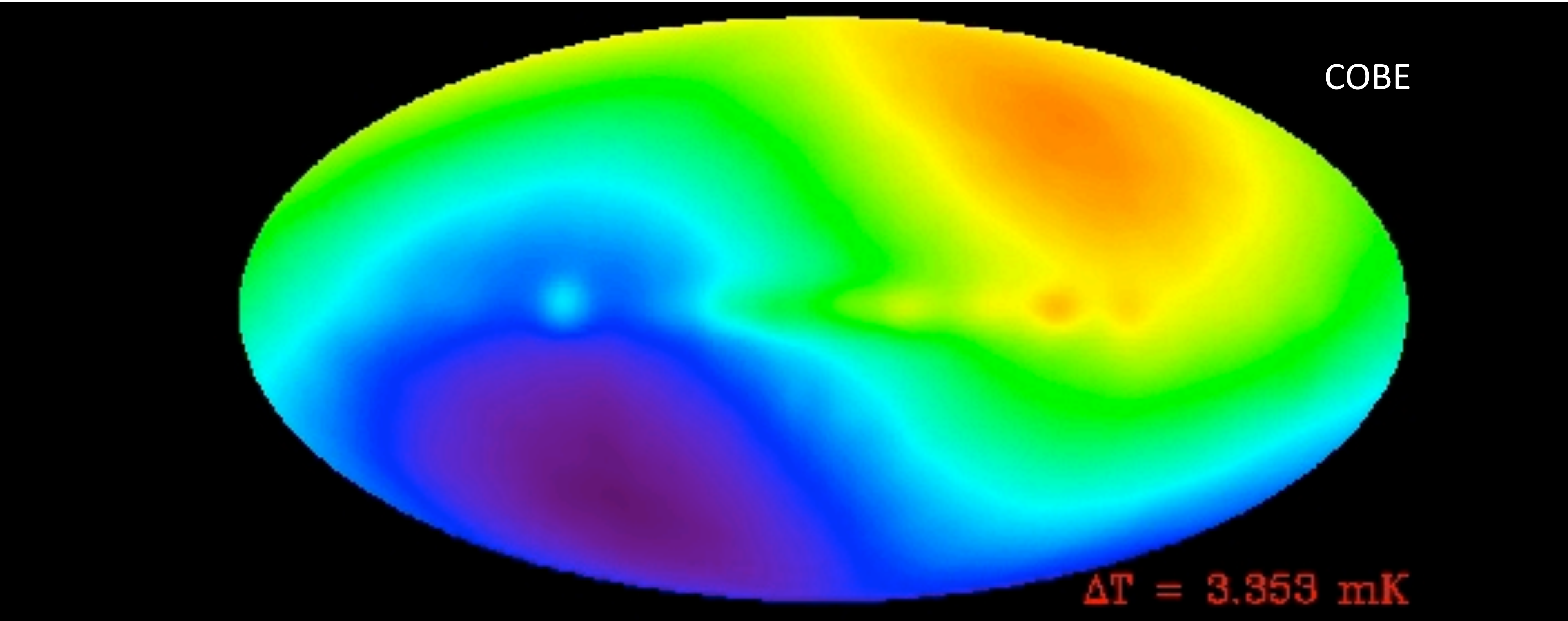
The monopole

- The CMB has a black body spectrum with average temperature of T=2.725±0.002 K (COBE, Mather et al.)



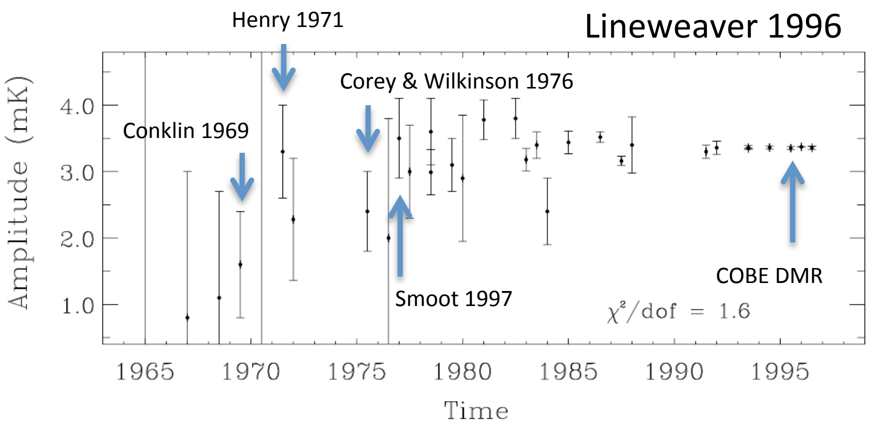
The dipole

- The motion of the sun w.r.t. the CMB reference system produces a dipole of $\Delta T = 3.3645 \ \pm 0.002$ mK (Planck 2015)



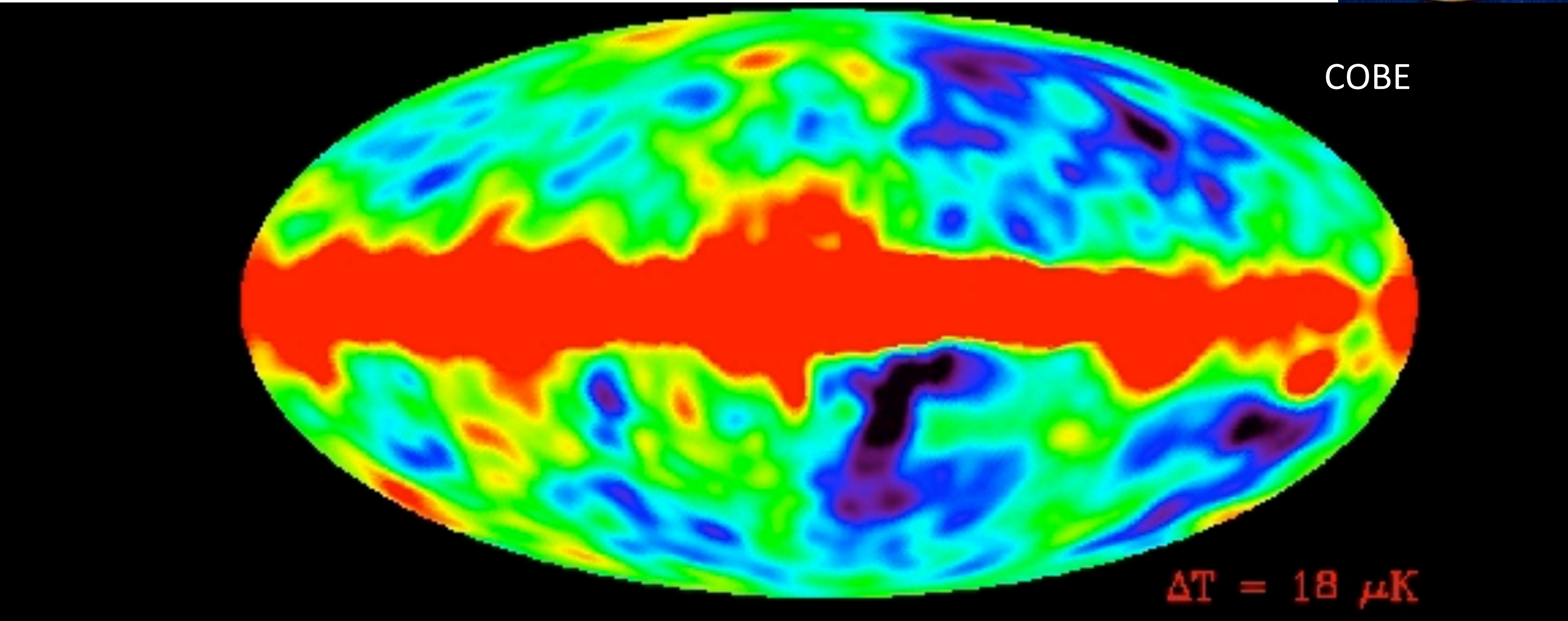
The dipole

- Detected shortly after Penzias and Wilson discovery.
- Due to the Doppler shift induced by the sun motion at v=370.5±0.2 km/s (Planck 2015)
- Earth motion w.r.t. sun produces a dipole 10 times smaller, v~30 km/s

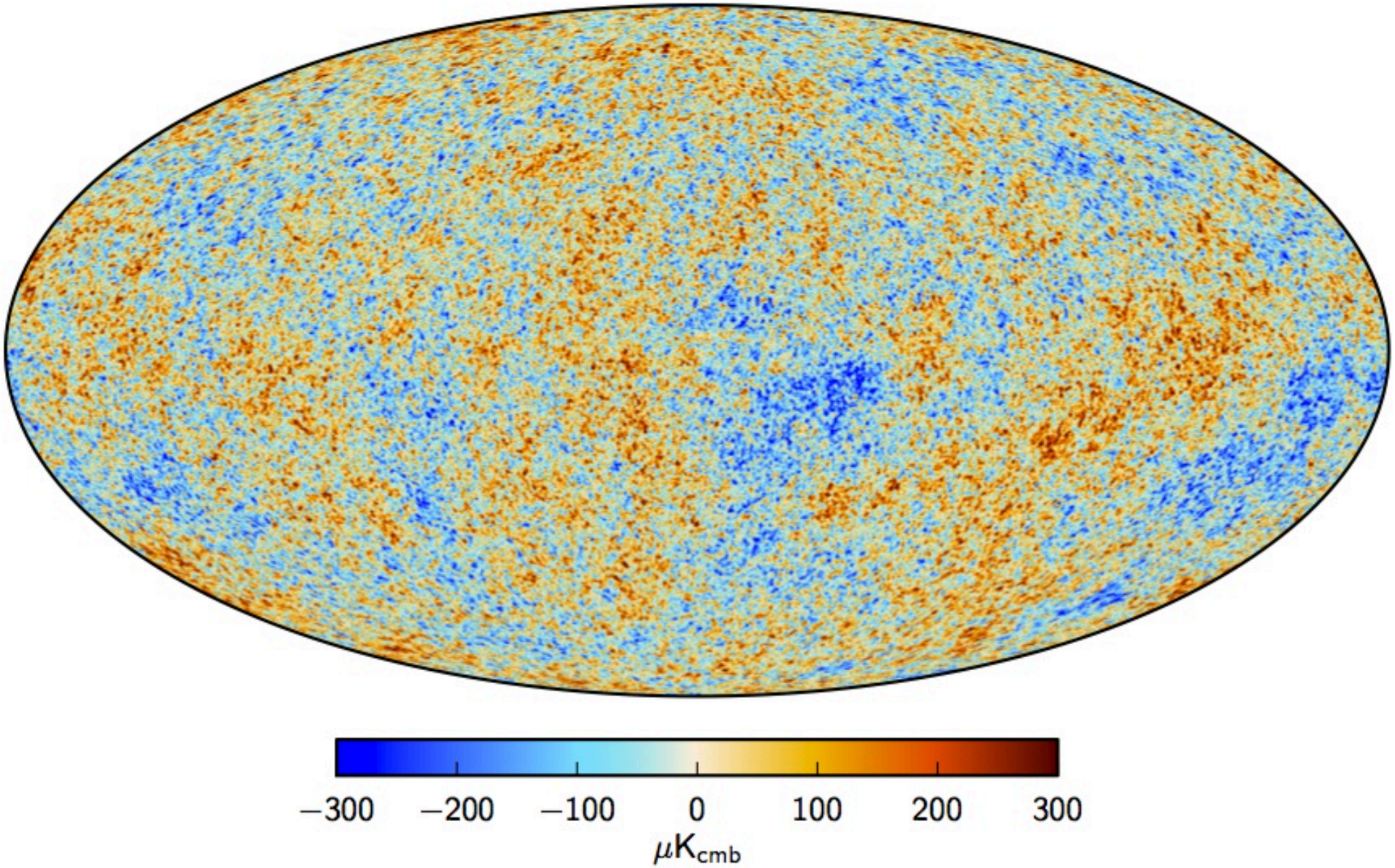


Anisotropies

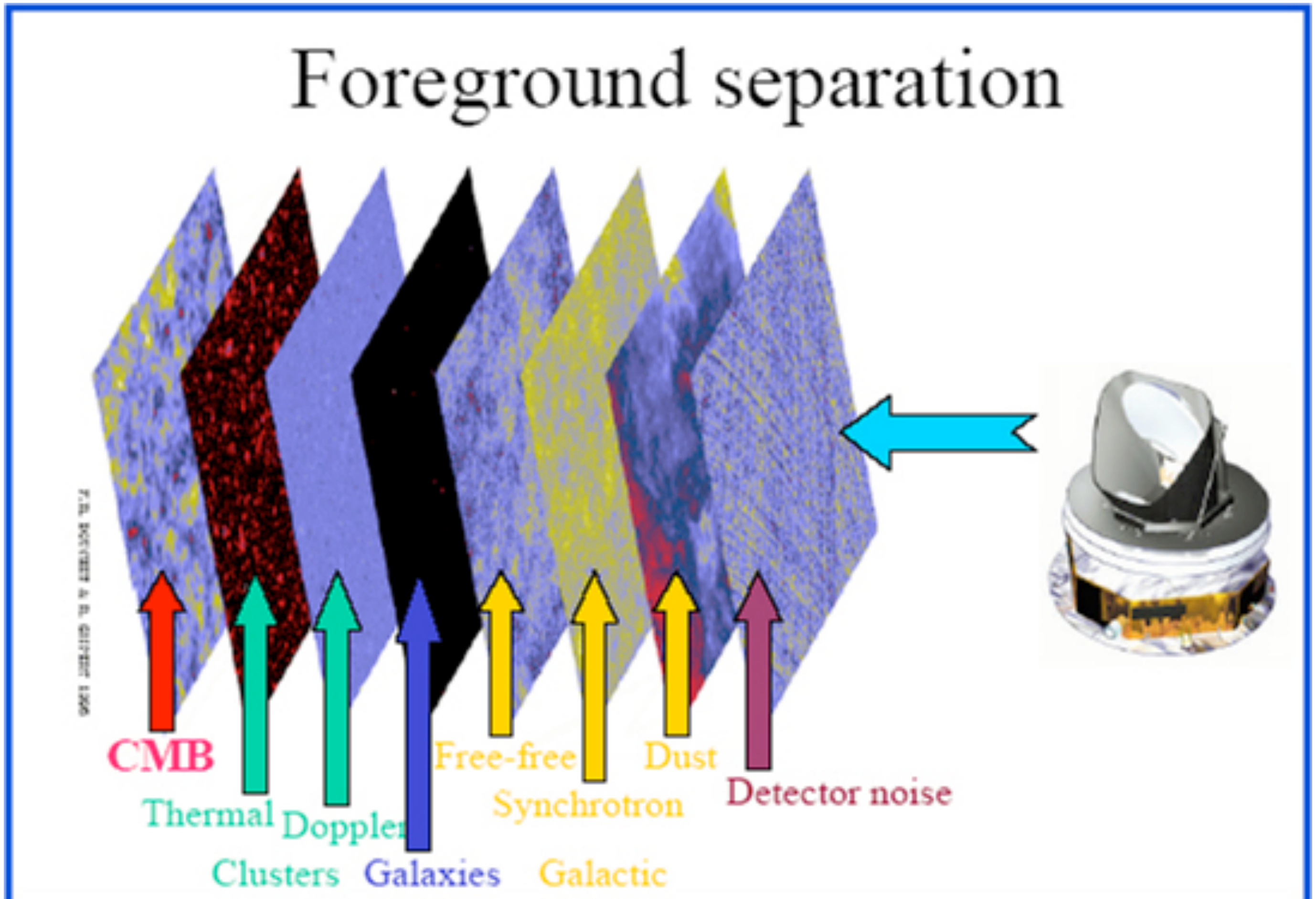
- At the μK level, CMB anisotropies (and foregrounds)!



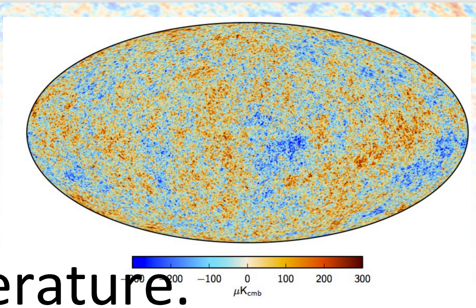
The Planck map



Not only CMB…


Foreground separation
CMB
Thermal
Doppler
Clusters
Galaxies
Free-free
Synchrotron
Dust
Galactic
Detector noise

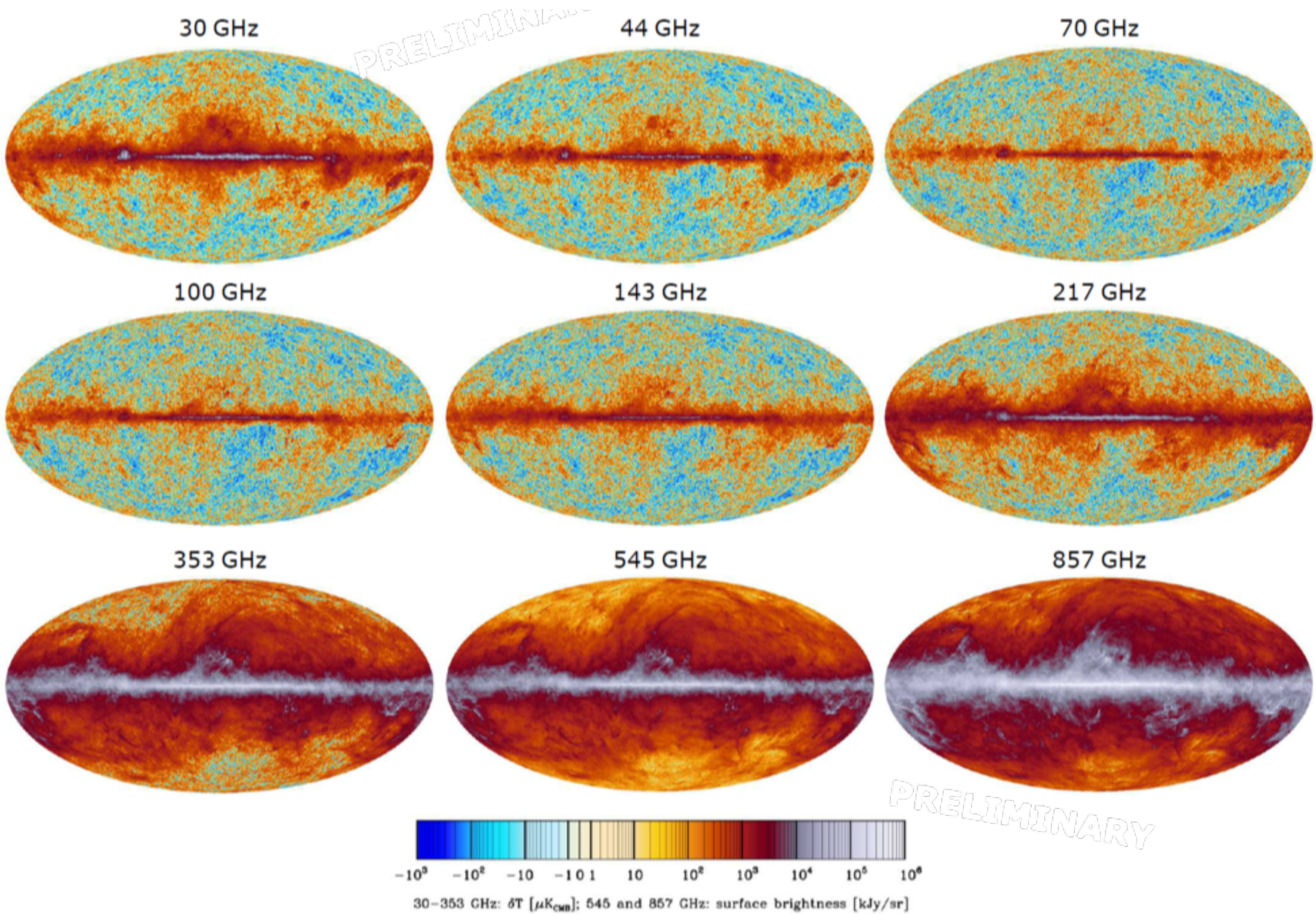
A note about units



- These maps are in units of thermodinamic temperature.
- Brightness to thermodynamic temperature K_{cmb} assuming a black body spectrum:

$$BB_\nu(T) = \frac{dE}{d\nu\, d\Omega\, dA\, dt} = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}$$

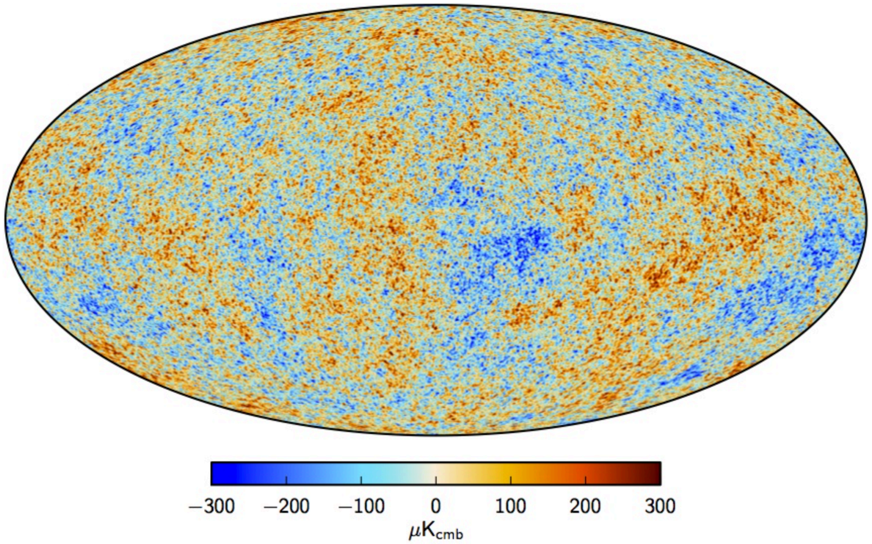
- In these units, the CMB has the same temperature at all frequencies, while foregrounds with different emission spectra have different thermodynamic temperatures at different frequencies



Planck 2015

Microwave sky

CMB STATISTICS

- CMB anisotropies are expected to be distributed as a gaussian random field.
- We cannot theoretically predict the value of the temperature in the pixels, but only predict their statistical properties.
- 

- A gaussian distribution is fully characterized by a mean and variance. All higher odd moments are 0, even moments can be written in terms of the variance (Wick's theorem)

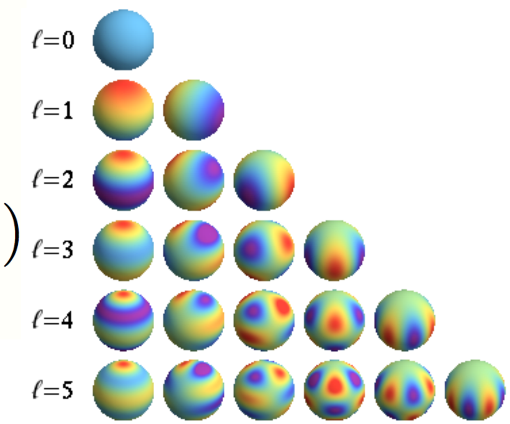
Spherical harmonics

- We can decompose the temperature maps in spherical harmonics.
- SH are a horto-normal basis of functions on the sphere. They are the eigenfunctions of the angular part of the Laplace operator in spherical coordinates.
- They are a hortogonal and complete basis.

$$\nabla^2 Y_\ell^m = -[l(l+1)]Y_\ell^m$$

$$\int d\hat{\mathbf{n}} Y_\ell^{m*}(\hat{\mathbf{n}}) Y_{\ell'}^{m'}(\hat{\mathbf{n}}) = \delta_{\ell\ell'}\delta_{mm'}$$

$$\sum_{\ell m} Y_\ell^{m*}(\hat{\mathbf{n}}) Y_\ell^{m}(\hat{\mathbf{n}}') = \delta(\phi - \phi')\delta(\cos\theta - \cos\theta')$$



- Complex. Conjugation:

$$Y_\ell^{m*} = (-1)^m Y_\ell^{-m}$$

- Characterized the degree (multipole) ℓ and the order m.
- $\ell \sim \pi/\theta$, with the θ angular separation in the sky.
- For each ℓ, $-\ell <= m <= \ell$. There are 2l+1 m-modes for each l.
- The projection on the m-modes depends on the reference system.

Decomposition in spherical harmonics

- Decompose the fractional temperature variation in spherical harmonics

$$\frac{\Delta T}{T}(\hat{n}, \vec{x}, \eta) = \Theta(\hat{n}, \vec{x}, \eta) = \sum_{\ell m} \Theta_{\ell m}(\vec{x}, \eta) Y_\ell^m(\hat{n})$$

Line of sight

Position in the sky (us ,x=0)

Conformal time (us, $\eta=\eta_0$) $\eta \equiv \int dt/a$

Also often called $a_{\ell m}$ in the literature

- Applying the orthogonality of spherical harmonics:

$$\Theta_{\ell m}(\vec{x}, \eta) = \int_\Omega d\hat{n}\, \Theta(\hat{n}, \vec{x}, \eta) Y^*_{\ell m}(\hat{n})$$

- In the simplest models of inflation, $\Theta(\hat{n})$ is a gaussian random field. Then, $\Theta_{\ell m}$ are statistically independent and randomly distributed, each described by a gaussian distribution.

Cl's in theory

- To characterize the statistical properties of a gaussian random field, we can calculate the mean and the variance of the field. For the CMB, the mean of the anisotropies is zero (by definition). The variance can be calculated either as the 2-point correlation function in real space, or equivalently, as the **angular power spectrum in harmonic space**.

$$< \Theta_{\ell m} >= 0 \qquad < \Theta_{\ell m}\Theta_{\ell' m'} >= \delta_{\ell\ell'}\delta_{mm'}C_{\ell}$$

- <> are ensemble averages over many realizations of the sky.
- Because of isotropy, $\Theta_{\ell m}$ with same ℓ and different m are extracted from gaussian distribution with the same variance C_ℓ

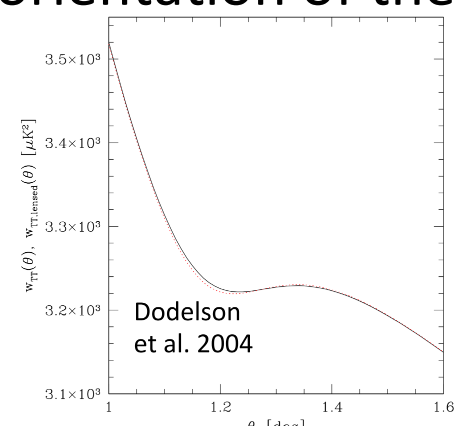
Cl's and 2-point correlation function

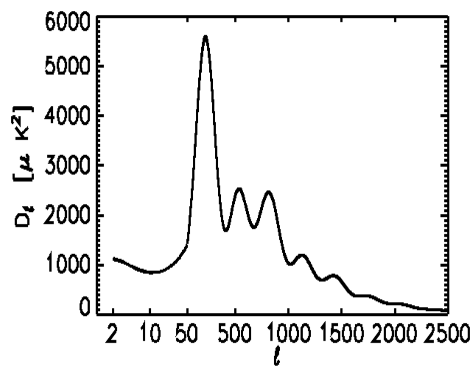
- We can relate the angular power spectrum to the 2-point correlation function in real space using the Legendre polynomials and the addition theorem:

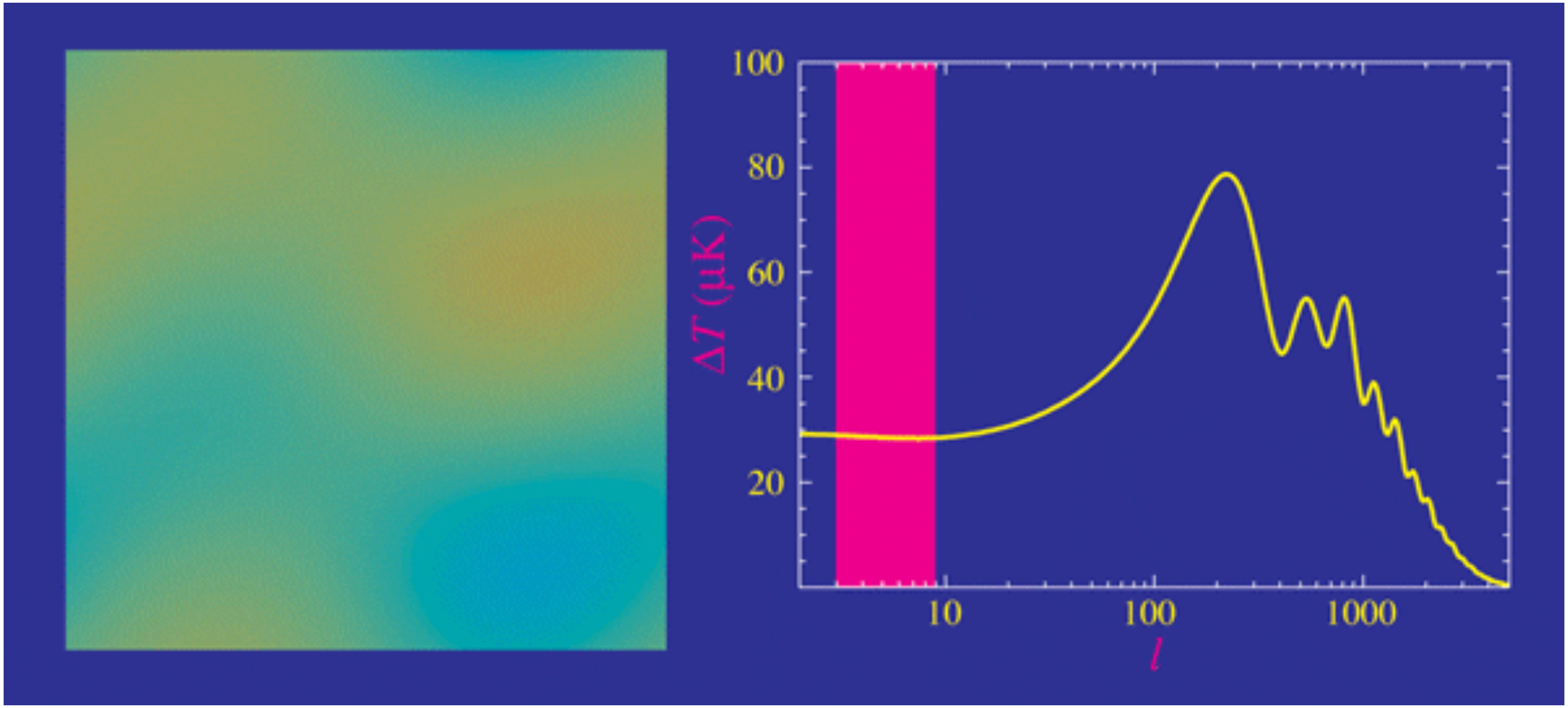
$$\sum_m Y^*_{\ell m}(\mathbf{n}_i) Y_{\ell m}(\mathbf{n}_j) = \frac{2\ell+1}{4\pi} P_\ell(\hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}_j)$$

$$\langle \Theta_i \Theta_j \rangle = \sum_\ell \frac{2\ell+1}{4\pi} C_\ell P_\ell(\hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}_j)$$

- Because of isotropy, the two-point correlation function depends only on the angular separation in the sky θ, not on the orientation of the separation.







Each of these maps are extracted from gaussian distributions with 0 mean and variance given by the Cl in the corresponding pink band. There is an infinite number of possible realizations.

An estimator for the Cl's

$$< \Theta_{\ell m} >= 0 \qquad < \Theta_{\ell m}\Theta_{\ell' m'} >= \delta_{\ell\ell'}\delta_{mm'}C_\ell$$

- We only observe one universe=> ensemble average not possible.
- Because of isotropy, all the m-modes $\Theta_{\ell m}$ with the same ℓ have the same theoretical C_ℓ. An estimator of C_ℓ is then:

$$\hat{C}_\ell = \frac{1}{2\ell + 1}\sum_m \Theta_{\ell m}\Theta^*_{\ell m}$$

Cosmic Variance

- The expected value is <Ĉl>=Cl
- Since we only have 2l+1 samples for each l, there is an intrinsic uncertainty!

$$\hat{C}_\ell = \frac{1}{2\ell+1}\sum_m \Theta_{\ell m}\Theta^*_{\ell m}$$

$$\frac{\sigma^2_{C_\ell}}{C_\ell^2} = \frac{\langle(\hat{C}_\ell - C_\ell)(\hat{C}_\ell - C_\ell)\rangle}{C_\ell^2} = \frac{\langle\hat{C}_\ell\hat{C}_\ell\rangle - C_\ell^2}{C_\ell^2}$$

$$= \frac{1}{(2\ell+1)^2 C_\ell^2}\langle\sum_{mm'}\Theta^*_{\ell m}\Theta_{\ell m}\Theta^*_{\ell m'}\Theta_{\ell m'}\rangle - 1$$

$$= \frac{1}{(2\ell+1)^2}\sum_{mm'}(\delta_{mm'} + \delta_{m-m'}) = \frac{2}{2\ell+1}$$

For a gaussian field, Wick's theorem says that any N-point (N even) statistics can be written as a function of the 2-point correlation function

$$\sigma^2_{C_\ell} = \frac{2}{2\ell+1}C_\ell^2$$